Gordon McKay Laboratory
Harvard University
Cambridge, Massachusetts

Semi-Annual Status Report on

Grant NGR-22-007-056

Theoretical and Experimental Investigations of Antennas and Waves in Plasma

For the period

September 1, 1966 through February 28, 1967

March 1, 1967

Prepared for
Grants and Research Contracts Division
Office of Space Science and Applications
National Aeronautics and Space Administration
Washington, D. C. 20546

Theoretical and Experimental Study of Finite Cylindrical Antennas in a Plasma Column - C. Y. Ting and B. Rama Rao.

By means of the Wiener-Hopf technique, the reflection coefficient of the transmission current at the end of a dielectric-coated antenna can be expressed in a single integral form. This result, when used with the solution of an infinite dielectric-coated cylindrical antenna, yields the input admittance and the current distribution of a long dielectric-coated antenna. It is found that unlike the locus of the admittance of a bare cylindrical antenna, which converges to a point as the antenna gets longer and longer, the locus of the input admittance of the dielectric-coated antenna becomes a circle. Also, due to the reflection of the transmission current back and forth a standing wave with wavelength equal to that of the surface wave is formed along the antenna.

An experimental study of the plasma-coated antenna is in progress. It has been found that the microwave-cavity perturbation method is the best way to measure the electron density and the effective collision frequency. The measurement of current distribution and input admittance is in progress.

Experimental and Theoretical Investigations on Plasma coated Antennas - B. Rama Rao and C. Y. Ting.

Extensive experimental measurements have been made to determine the current-distribution and impedance characteristics of plasma-coated cylindrical dipole antennas with lengths comparable to the free space wavelength. Measurements were made at frequencies both above and below the plasma frequency. When the excitation frequency of the antenna is above the plasma frequency, the plasma sheath appears to reduce the electric length of the antenna, causing the current distribution to become more uniform. The experimental results show a "reasonable" degree of qualitative agreement with the theoretical results obtained by C. Y. Ting assuming a lossless cold-plasma model (hydrodynamic approach). In the vicinity and below the plasma frequency, the current distribution of the antenna undergoes a very marked change - the hydrodynamic theory seems unable to account for this phenomenon. An alternative approach using the moment equations of the Boltzmann equation (to take into account the temperature effect) seems necessary.

By far the most interesting effect noticed was the sharp increase in the input resistance of a short dipole antenna as the electron density was increased. The input impedance of a short antenna $(\frac{\lambda_o}{32})$ is mainly capacitive, the measured input resistance being only 4 ohms. As the electron density (discharge current) was steadily increased, the input resistance of the antenna rose sharply to 400 Ω and then fell off to 200 Ω and remained steady thereafter, with further increase in the plasma frequency. This "resonance" phenomenon occurs below the measured plasma frequency; the resonance line apprears quite broad. A suitable theoretical explanation for this resistance peaking is being sought by taking into account the resonances in the warm plasma column. Strong resonances in the plasma column could absorb the electromagnetic energy supplied to the antenna thereby causing its radiation resistance to increase sharply. The resonance line observed was quite broad, indicating the possibility of some type of collisionless damping. The possibility of using short dipole antennas for studying such plasma column resonances is being investigated.

Measurements are now being made to study the effect of the electron temperature in the antenna characteristics. These investigations will be made in the vicinity of the plasma frequency, where the compressibility effects of the plasma should have their maximum impact.

The experimental results will be reported in a forthcoming technical report.

Plasma Sheath Investigations Using a Microwave Cavity Technique - B. Rama Rao and L. D. Scott.

The cavity perturbation method has been widely used as a diagnostic technique for studying plasma properties. The accuracy of this method depends largely on the assumed electron-density profile distributions used in the calculations. The profile distribution commonly used in this type of measurement is the Schottky diffusion profile of the type $J_o(2.404 \frac{r}{r_p})$, when r_p is the radius of the plasma column. Many investigators fail to realize that this type of profile is only valid at high pressures when the mean free path of the electrons is small compared to the radius r_p of the plasma column; substantial errors can result if the same profile is used for measurements at low pressures or when the plasma column is very thin.

The purpose of this study is to use a multi-mode cavity method so that the plasma diagnosis can be extended over a much wider range of pressures. Subsequently this method will be employed to make an experimental study of the plasma sheath problem - where the radial density distribution will be studied for a wide range of pressures and electron temperatures. A similar method has been suggested by Stewart[4], but it has not been used for the specific purpose of diagnostic and sheath investigations.

To simplify the analysis of the problem, three different pressure regimes will be considered.

(1) High pressure regime, (pressure > 1 mm of Hg: mean free path << r_p; ambipolar diffusion type regime). The differential equation for the electron density with both ambipolar diffusion and volume recombination taken into account may be written in the form[1]

$$\frac{d^2n}{dx^2} + \frac{1}{x}\frac{dn}{dx} + \frac{\nu r_p^2}{D_a} n - \frac{\beta r_p^2}{D_a} n = 0$$

where r_p = plasma tube radius, D_a = ambipolar diffusion constant, ν = coefficient of ionization, and variable $x = r/r_p$. Solution to this equation is of the form

$$n = n_o + \sum_{\ell=1}^{\infty} - \frac{1}{(2\ell)^2}\left[\frac{\nu r_p^2}{D_a} a_{2(\ell-1)} - \frac{\nu r_p^2}{D_a} \sum_{k=0}^{\ell-1} a_{2k} a_{2(\ell-k-1)}\right] x^{2\ell}$$

Hence in this regime the electron density profile is of the form

$n = n_o - n_2 x^2 + n_4 x^4$. Only the first three terms need be considered since the above series is rapidly convergent. The next step in the experimental technique is to measure the shift in the resonant frequency using three different cavity modes - with different electric field distributions along the radial directions. For the right circular cylindrical cavity used in the experiments the three modes chosen were the TM_{020}, TM_{120}, and the TM_{210}. These have only E_z components which vary respectively as $J_o(\frac{5.52r}{r_p})$, $J_1(\frac{7.016r}{r_p})$, and $J_2(\frac{5.136r}{r_p})$. By measuring the frequency shifts of these three modes, the profile coefficients n_o, n_2 and n_4 can be calculated. n_o is the electron density at the axis of the plasma column. Additional checks were made noting the TM_{012} and the TM_{212} modes. The measurements were made on a hot-cathode discharge tube using Helium.

(2) Intermediate pressures - longer mean free paths regime. In this range, Tonks and Langmuir[2] have shown that the density profile can be described approximately by the series

$$n = n_o(1 + x + x^2 + \dots)$$

This is true only when the ion generation is proportional to the electron density.

(3) Very low pressures - long mean free paths regime. The plasma sheath equation for this use has been solved by J. V. Parker,[3] when the ion generation is proportional to the electron density. The profile distribution is quite complicated in this regime and can be expressed in terms of two quantities:

$$M = \frac{\int_o^{r_p} n(r)\, r^3 dr}{r_p^2 \int_o^{r_p} n(r)\, r dr}$$

and
$$\beta^2 = \frac{n_o e^2}{\varepsilon_o kT} (r_p/s_w)^2$$

So far the experimental investigations have been confined only to the high pressure regime. They reveal that the distribution profile is approximately of the Schottky type at pressures above 1 mm. Agreement is poorer as the pressure decreases. Investigations are now being made at lower pressures ranging from 50μ to 10^{-3}μ of Hg.

A technical report on this problem is under preparation. A theoretical study of the collisionless plasma sheath equation for a spherical geometry is being attempted by Rama Rao.

1 "Volume recombination in the presence of ambipolar diffusion in a gas-discharge plasma", V. I. Solunski and B. L. Timan, Soviet Physics - Tech. Phys., Vol 9, No. 2, Aug. 1964, p. 207.

2 L. Tonks and I. Langmuir, Physical Review, Vol. 34, 1929, p. 876.

3 J. V. Parker, Physics of Fluids, Vol. 6, 1963, p. 1657.

4 "Microwave Measurements of Electron Density Profiles in Plasmas", G. E. Stewart and Z. A. Kapriellan, Phénoménes d'ionisation dans les gaz, (Paris 1963), Vol. IV, July.

Transmission Lines in Plasmas - William A. Saxton and Y. S. Yeh.

This study concerns the interaction between an open, balanced two-wire transmission line and a slightly ionized gaseous plasma. After a fundamental investigation of TEM transmission line waves through the plasma several possible applications will be considered, including use of a transmission line in plasma diagnostics and the feasibility of coupling plasma RF noise directly to such a line which could serve as a coupling element between the plasma and a radiating antenna in a penetrating aid device.

In the initial experimental set-up the transmission line passes through the positive column of a hot-cathode dc discharge-tube plasma in a direction transverse to the discharge. TEM waves propagating along the line experience a continuous variation of electron density. The effects of various density profiles on the transmission and reflection coefficients are being analyzed and solved rigorously by the integral equation method and through numerical computation.

If one assumes a Bessel-function radial electron density distribution with a plasma frequency w_p = 900 MHg at the center of the tube, corresponding to an electron density $n_o = 10^{10}$/c.c., the expected phase shift at $w_p/w = 0.9$ would be 24^o, where w is the electromagnetic frequency. On the other hand, the commonly used phase integral method would yield a shift of 19^o. At $w_p/w = 1$, the two are given by 31^o and 22^o respectively. Such a typical discrepancy comes mainly from the continuous multiple reflections in the plasma which are not negligible if the variation in density profile is not gradual compared to the RF wavelength.

Two baluns have been constructed to facilitate balanced measurements on a slotted coaxial line. The effects of the balun junction are determined by Deschamp's method so that they can be represented by a scattering matrix. Solution of the integral equation provides another scattering matrix for the plasma junction and completes the description of the experimental set-up.

Among the further steps in this investigation are the assumption of different plasma electron-density profiles ~~and their~~ effects on the transmission line parameters. Also planned is a comparison between experimental and theoretical scattering matrices and values of phase shift and attenuation through the discharge.

Brush-Cathode Discharges - William A. Saxton.

Until recently it has been very difficult to produce electrically quiet laboratory plasmas without instabilities and striations. However, the initial work by Persson at the National Bureau of Standards on cold-cathode "brush" discharges[1] unveiled the possibility of generating well-behaved and controllable gaseous plasmas for research work.

In their initial forms, brush cathodes consisted of large numbers of small diameter metallic needles, similar to sewing needles, mounted on flat circular disks. Such a configuration sealed in an inert gas discharge tube, and used in conjunction with a similar unit at a lower electrical potential, was found to generate a large uniform electron beam with a corresponding negative glow which had a longitudinal dimension one or two orders of magnitude layer than for a normal hot- or cold-cathode discharge. This negative glow was essentially field-free and recombination-dominated, making it a practically uniform plasma.

Although the brush-cathode has created a great deal of interest and study, there are many conflicting theories on the nature of its operation. In addition, not enough comparative experimental data are available to analyse the critical parameters in brush-cathode construction, nor to predict and evaluate the properties of the plasmas that it produces. This investigation is an attempt to shed some light on the brush-cathode phenomenon through an organized and detailed experimental study.

Three pairs of brush configurations have been built to date for comparison; a fourth pair is under construction. Each pair is circular cylindrical with a nominal diameter of $2\frac{1}{4}$ inches and lengths varying from 2-3 inches. Cathodes in one pair consist of over 1300 1-5/6" stainless-steel sewing needles brazed on a stainless-steel backplate. These cathodes will be compared to so-called "inverse" brush cathodes in which over 1300 0.027" holes will be dulled $1\frac{5}{16}$" deep into a stainless-steel cylinder. A third pair of cathodes is of the inverse-brush type made of Monel alloy, but with much fewer holes than the preceding, and of larger diameters - 3 mm diameter holes uniformly spaced on 5 mm centers drilled $1\frac{5}{16}$" deep. The fourth pair consists of circular cylinders with smooth flat faces also made of Monel.

Each pair of cathodes will be studied in turn using a glass test chamber attached to a vacuum system. The test chamber will provide for separations of one to six inches between the two cathodes in the discharge. Five Langmuir probes have been built for electron density and temperature measurements in the discharges. Three probes will be radially moveable in addition to being regularly spaced in the discharge column so that the entire discharge can be probed. Volt-ampere characteristics of the discharge will also be measured for each discharge condition.

Several discharge parameters will be considered for each pair of cathodes, including gas pressure, discharge current and voltage, election temperature and density, and collision frequency. In addition, electron density profiles will be determined, cathode spacing will be varied, and different types of gas will be used. By using an on-line real-time computer program developed for this experiment it should be possible to reduce the vast amount of Langmuir-probe data that will be obtained with the several degrees of freedom in the experiment. The results should be an exhaustive compilation of brush-discharge data.

1 K.B. Persson, "The Brush Cathode Plasma - A Well-Behaved Plasma", NBS Report 8452, U.S. Dept. of Commerce, National Bureau of Standards, Boulder Laboratories, Boulder, Colorado, Sept. 1964.

Acoustic Effects in Plasmas - William A. Saxton.

Fully calibrated moving armature and solid-dielectric transducers were used to detect and measure acoustic waves in the 200-2000 c/s range in an effort to correlate sound waves with so-called travelling striations which often occur in gas discharges. Although many data have been taken thus far in various laboratory plasmas, no conclusions are yet possible. Complicating the attempt to tie the electrical and acoustical effects together is the apparent presence of standing acoustic waves in the particular geometrics of the discharge tubes.

One interesting application for the acoustic waves which accompany the discharge striations is in modulating electromagnetic waves which impinge on a gaseous plasma column. Recent theoretical and experimental work indicates a significant scattering of electromagnetic waves propagating through a plasma which has a periodically-varying electron density transverse to the direction of propagation. Such an effect could be useful in varying the lobe structure of microwave antennas, for example, if the plasma properties were controllable.

Density variations which result from moving striations are usually time-variant and unstable. Another way to use the modulation technique is to propagate sound waves into a quiet plasma using an acoustic transducer of the type developed by Saxton.[1] This perturbation method has been shown to produce definite electron density and collision-frequency variations in the positive column of a dc discharge at audio-frequencies[3] and does lend itself to predictable and reliable control of these plasma parameters both in amplitude and frequency. Construction of an experiment to investigate the EM-wave modulation effect has begun and a discharge tube has been completed in which it will be possible to study the scattering of electromagnetic waves by plasma acoustic waves created internally by striations, externally by transducer inputs, and by artificially stimulated ion acoustic waves.

A third phase of this work is directed toward an experimental study of acoustic wave amplification in gaseous plasmas. Ingard[3] at M.I.T. has laid the theoretical groundwork for such an investigation by showing that, under certain conditions, the electrons in a weakly ionized plasma heat the neutral-gas component and either generate or amplify sound waves. An experiment has been designed which should serve to explore the amplification possibility.

Since the magnitude of any acoustic amplification is increased by enlarging the ratio of electrons temperature to neutral-gas temperature[2] it is planned to immerse part of the plasma in cryogenic surroundings. For this purpose a U-shaped discharge tube is planned with the bottom part of the "U" immersed in liquid helium. A brush cathode discharge will be used to assure freedom from any acoustic waves generated by the plasma itself. Sound waves will be forced into the cryogenic portion of the plasma using a moving armature transducer. After the sound passes through the cryogenic interaction region, where most of the amplification should take place, it will be detected and measured at the other end of the U-tube with a solid-dielectric microphone. All components in the discharge-tube itself, are fully bakeable and consistent with proper vacuum techniques.

Typical conditions in a helium discharge include a pressure of one Torr, a neutral gas temperature of $10^{\circ}K$ in the cryogenically-cooled portion of the tube, an electron temperature equal to $1000^{\circ}K$, and an electron density of 5 x 10" electrons/c.c. In a U-shaped $1\frac{1}{2}$" diameter tube of the proper length and geometry, under these conditions it should be possible to achieve gains of at least 5-10 db. Discharges with higher electron temperatures, such as conventional hot- or cold-cathode types, should yield significantly higher gains.

1 Saxton, W. A., "Transducers for Exciting and Detecting Acoustic Waves in Discharge-Tube Plasmas", Journal of the Acoustical Society of America, Vol. 38, No. 6, Dec. 1965.

2 Saxton, W. A., "Excitation of Acoustic Waves in Plasmas", Radio Science, NBS Journal of Research, Vol. 69D, No. 4, April 1965.

3 Ingard, U., "Acoustic Wave Generation and Amplification in a Plasma", Physical Review, Vol. 145, No. 1, May 1966.

Theoretical Studies on an Antenna Immersed in a Plasma - A. D. Wunsch.

Numerous attempts at ionospheric exploration are being made using satellite-borne antennas. Generally, the intention in such experiments is to determine the constitutive parameters (electron density, magnetic field strength, temperature, etc.) of the ionosphere through the use of impedance measurements on the antennas. The purpose of the present research is to establish a theory relating these impedance measurements to the ionospheric parameters.

One aspect of the problem which has been completed, treats the ionosphere as a cold, homogeneous, anisotropic medium. The plasma is characterized by a dielectric tensor, and the antenna is regarded as a strip of electric current. The radiation resistance of the antenna has been determined with a current perpendicular to the magnetic field of the surrounding medium. A paper reporting the results of this investigation has been accepted for publication by the Canadian Journal of Physics and will appear shortly.

Another aspect of this research which is close to completion treats the plasma as a homogeneous, isotropic medium of finite temperature. In this investigation expressions have been obtained not only for the impedance of a cylindrical dipole antenna immersed in plasma but also for the distribution of current along the antenna. At present, the validity of using various simple, assumed current distributions as a means for obtaining values of the antenna impedance is being examined.

Coupled Monopoles in a Plasma - J. C. Robertson.

Extensive theoretical and experimental work has been completed on the properties of parallel coupled monopole antennas in a dissipative dielectric medium. Plans are in progress to investigate experimentally and theoretically the related problem of two coupled monopoles immersed in a plasma. In particular, the distributions of current and charge along the antennas and their admittances will be investigated.

The staff now working on this grant consists of Professor B. Rama Rao, Dr. William A. Saxton and three part-time students C. Y. Ting, A. D. Wunsch, and J. C. Robertson.

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Submitted by

Ronold W. P. King

Ronold W. P. King, Director
NASA Grant NGR-22-007-056